MMATH18-201

M.A./M.Sc. Mathematics, II Semester
Module Theory
Assignment Topic
Left Module, Exact Sequence, Free Module, Semisimple Module and Chain Conditions — 50 Marks

1. Let R be a ring with unity. Define a left R-module. For a non-empty subset S of M prove that $< S >= \{\sum_{i=1}^{n} r_i x_i : r_i \in R, x_i \in S, n \in \mathbb{N}\}$. 10 Marks

2. Let $f: M \to M$ be an endomorphism of an R-module M such that $f^2 = f$. Then prove that $M = Im\, f \oplus Ker\, f$. 10 Marks

3. Let M be a free R-module with basis X, then prove that each summand R_x is isomorphic to left R-module ${}_R R$, and $M = \bigoplus_{x \in X} R_x$. 10 Marks

4. Prove that M is semisimple if and only if $M = scole\ of\ M$. Give an example of a module which is not semiisimple. 10 Marks

5. Let M be an R-module. Then prove that M is Noetherian if and only if every submodule of M is finitely generated. . 10 Marks

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M. Sc. (Mathematics) Part-I Semester II
Assignment
MMATH18-202 Topology
Marks-50

Note- Attempt each question.

1. Define Topological Space and its basis. Show that the intersection of family of topologies is topology on X. What about the union of two topologies ?
2. Let X and Y be two topological spaces and $f : X \to Y$ be a continuous map. Prove that

   (i) $f^{-1}(F)$ is closed in X , for every closed set $F \subseteq Y$.

   (ii) $f(\bar{A}) \subseteq \overline{f(A)}$ for every set $A \subseteq X$.

   (i) $\overline{f^{-1}(B)} \subseteq f^{-1}(\bar{B})$, for every $B \subseteq Y$.

3. (i) Prove that if a non-empty subset X of the real line R is an interval then it is connected.

   (ii) Show that if A is connected subset of X then $\bar{A}$ is also connected.

4. (i) Show that a closed mapping $f : X \to Y$ is proper if $f^{-1}(y)$ is compact for each $y \in Y$

   and hence or otherwise, prove that the product of two compact spaces is compact.

   (ii) Is the interval (0,1] countably compact in the Euclidean real line? Justify.

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M. Sc. (Mathematics) Part-I Semester II
Assignment
MATH18-203 Functional Analysis
Marks-50

Note- Attempt each question. Each question carries equal marks.

Q1. State and prove Riez's Lemma.

Q2. Show that every finite dimensional subspace Y of a normed space X is complete.

Q3. Prove that if a normed space X is finite dimentional, then every linear operator on X is bounded.

Q4. Prove that if Y is any closed subspace of a Hilbert space H, then $H = Y \oplus Z$ where $Z = Y^{\perp}$

Q5. Prove that a bounded linear operator T from a Banach space X onto a Banach space Y is an open mapping.

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M. Sc. (Mathematics) Part-I Semester II
Assignment
MMATH18-204 Fluid Dynamics

Total Marks-50

Attempt all the five questions.

1. Explain the continuum model. Discuss the two criterion of continuum model. Describe the classification of fluid. [3+3+4]
2. Describe the difference between stream line, path line and streak lines. Discuss the differentiation following the fluid motion. Introduce the velocity potential and vorticity vector. Prove that stream lines and equipotential lines intersect orthogonally. [3+2+3+2]
3. Derive the equation of continuity and boundary surface condition. [5+5]
4. Derive the Euler's equation of motion and discuss the integration of Euler's equation of motion. [5+5]
5. Define stream function, complex potential, line source and line doublets. Describe the applications of Milne's Thomson Circle Theorem and Blasius's Theorem with suitable examples. [4+3+3]